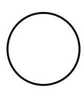
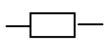
Power System Analysis

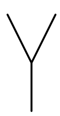
The one line diagram

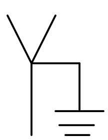
Modern power systems are invariably three phase. In a balanced three phase systems is always solved as a single phase circuit composed of one of the three lines and a neutral return and this is sufficient to give a complete analysis. Such a simplified diagram of an electric system is called a one – line diagram.
Figure 1 below shows the symbols for representing the components of a three phase power system.


Rotating Machine(Generator or Motor).


Circuit Breaker.


3-phase delta connection.


3-phase star connection (neutral ungrounded).


3-phase star connection (neutral grounded).

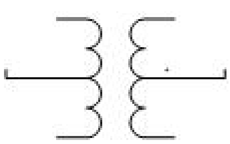

Two winding power transformer .

Figure-1-symbols of components of a 3-phase power system.
Figure-2 shows the one line diagram of a simple 3-phase power system.

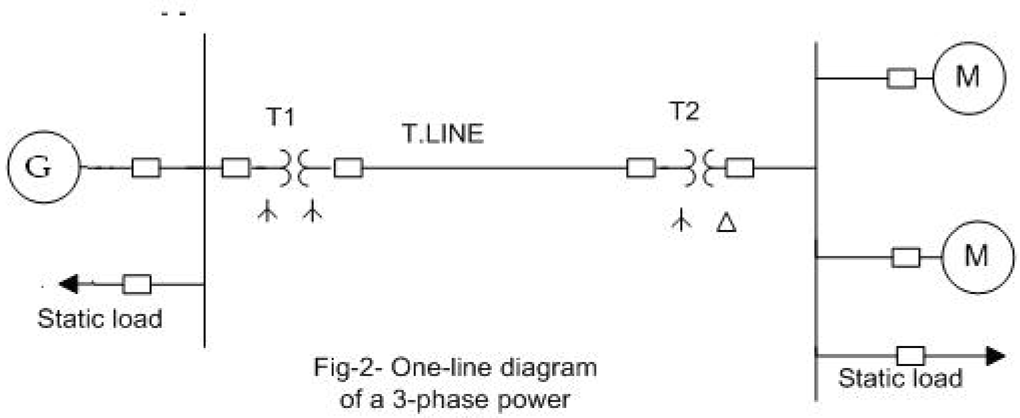


Fig-2- One-line diagram of a 3-phase power system

Notes

1. Generators neutrals are usually grounded through high resistances and somtimes through inductance coils in order to limit the flow of current to ground during a fault.
2. Most transformer neutrals in transmission systems are solidly grounded .

Per unit quantity

When making calculations on a power system network having two or more voltages levels, it is very cumbersome to convert currents to different voltage levels at each point where they flow through a transformer, the change in current being inversely proportional to the transformer turns ratio. In order to simplify these calculations we can use per unit system.
In this system a base quantities are assumed for each voltage level and the per unit quantities are calculated as follow:

$$\text{Per unit quantity} = \frac{\text{Actual quantity}}{\text{Base quantity}} \quad \text{--------------------------(1)}$$

The four electrical quantity (voltages, current, power, and impedance) are so related that selection of base values for any two of them determine the base values of the remaining two.
Usually base apperant power in megavoltamperes and base voltage in KV are quantities selected to specify the base values.

For single phase system or 3-phase on per phase basis , the following relationships hold:

$$\text{Base current} = \frac{\text{Base voltamperes}}{\text{Base voltage}} \text{ ------------------------------------(2)}$$

$$\text{Base impedance} = \frac{\text{Base voltage}}{\text{Base current}} \text{ --------------------------------------(3)}$$

$$\text{Per unit voltage} = \frac{\text{Actual voltage}}{\text{Base voltage}} \text{ --------------------------------------(4)}$$

$$\text{Per unit current} = \frac{\text{Actual current}}{\text{Base current}} \text{ ---------------------------------------(5)}$$

$$\text{Per unit impedance} = \frac{\text{Actual impedance}}{\text{Base impedance}} \text{ ----------------------------(6)}$$

Or

$$\text{Base current} = \frac{\text{Base KVA 1-phase}}{\text{Base voltage KV-ph}} \text{ ---------------------------------(7)}$$

$$\text{Base impedance} = \frac{\text{Base voltage Vph}}{\text{Base current Amp}} \text{ -------------------------------(8)}$$

$$\text{Base impedance} = \frac{(\text{Base voltage KVph})^2 \times 1000}{\text{Base KVA-ph}} \text{ ------------------(9)}$$

$$\text{Base impedance} = \frac{(\text{Base voltage KV-ph})^2}{\text{Base MVA -ph}} \text{ ------------------(10)}$$

$$\text{Per unit impedance} = \frac{\text{Actual impedance}}{\text{Base impedance}} \text{ -----------------------(11)}$$

If we choose base kilovoltamperes and base voltage in kv to mean kilovoltamperes for the total of the 3-phases base voltage for line to line, we find:

$$\text{Base current} = \frac{\text{Base KVA-3-ph}}{\sqrt{3} \times \text{Base KVA-line}} \text{ ---------------------------(12)}$$

For 3-phase

$$\text{Base impedance} = \frac{(\text{Base voltage KV})^2}{\text{Base MVA-3-ph}} \text{ ---------------------(13)}$$

<u>Note:</u>

In a 3-phase system, the per unit 3-phase kVA and voltage on the 3-phase basis is equal to the per unit per phase kVA and voltage on the per phase basis.

<u>Example1-1</u>: Consider a 3-phase wye – connected 50000 kVA, 120kV system. Express, 40000 kva three phase apperant power and 115 kv line t line voltage in per unit values on (i) 3-phase basis and (ii) per phase basis.

Solution;

(i) Three phase basis

Base kva = 50000 kva

Base kv = 120 kv (line to line)

$$\text{P.u kva} = \frac{40000}{50000} = 0.8$$

$$\text{p.u voltage} = \frac{115}{120} = 0.96$$

(ii) per phase basis

$$\text{Base kva} = \frac{1}{3} \times 50000 = 16667 kva$$

Base KV = $\frac{120}{\sqrt{3}}$ =69.28 KV

Per unit KVA= $\frac{40000/3}{16667}$=0.8

Per unit voltage =$\frac{115/\sqrt{3}}{69.28}$=0.96

Change of Base

Sometimes it is necessary to convert per-unit quantities from one base to another. The conversion formula for the impedance can be written as follow:

$$(Z_{pu})_{new} = (Z_{pu})_{old} \times \left(\frac{S_{b-new}}{S_{b-old}}\right) \times \left(\frac{V_{b-old}}{V_{b-new}}\right)^2 \text{.----------------------------------(14)}$$

Example1-2

The reactance of a generator Xg is given as 0.25pu based on generator nameplate rating of 18KV , 500MVA.The base for calculations is 20KV
And 800MVA.Find Xg on the new base.

Solution

$$\text{Xg-new}=0.25\times\left(\frac{100}{500}\right)\times\left(\frac{18}{20}\right)^2 =0.0405\text{pu}$$

<u>**Per unit impedance of transformer unit**</u>

The ohmic values of impedance of a transformer depend on whether they are measured on the high or low tension side of transformer
In the per unit system, the per unit impedances of a transformer is the same regardless of whether it is determined from ohmic values referred to the high tension or low tension side of the transformer.

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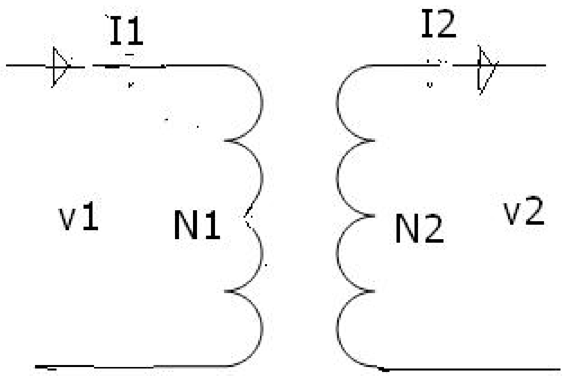


Z1= $\frac{N1^2}{N2^2}$ ohm ----------------(14) . Z1=$\frac{V1^2}{V2^2}$ ohm ------------------(15 ,

The impedance bases on the two sides of the transformer are from equation (13).

$Z1_{base} = \frac{KV1^2}{MVA_b}$ -------------------------------(16).

Where KV1 is the 1st side base voltage.

$Z2_{base} = \frac{KV2^2}{MVA_b}$ -----------------------------(17).

Where KV2 is the 2^{nd} side base voltage.

To prove that Z1pu=Z2pu

$$\frac{Z_{1b}}{Z_{2b=}} = \frac{\frac{KV_{1b}}{MVA_b}^2}{\frac{KV_{2b}{}^2}{MVA_b}} = \frac{V_{1b}{}^2}{V_{2b}{}^2} = \frac{N_1{}^2}{N_2{}^2}.$$

$$Z_{1pu} = \frac{Z_1\Omega}{Z_{1b}\Omega} = \frac{\frac{N_1{}^2}{N_2{}^2}Z_2}{\frac{N_1{}^2}{N_2{}^2}} = \frac{Z_2\Omega}{Z_{2b}\Omega} = Z_{2pu}$$

Example 1-3 A single phase transformer is rated 110/440, 2.5 kva, leakage reactance measured from low tension side is 0.06 Ω. Determine leakage reactance in per unit.

Solution

From the low tension side

Base KV1=110×10^{-3}

Base MVA= 2.5×10^{-3}

Z1 $_b$=$\frac{110\times10^{-3})^2}{2.5\times10^{-3}} = 4.84\Omega$ (base impedance on low tension side).

Z1pu=$\frac{Z_1\Omega}{Z_{1b}} = \frac{0.06}{4.84} = 0.0124pu$

From the high tension side

Base KV2=440 V

Base MVA=2.5×10^{-3}

Z2=$\frac{440}{110^2}^2 \times 0.06 = 0.96\Omega$

$$Z2_b = \frac{440 \times 10^{-3})^2}{2.5 \times 10^{-3}} = 77.44\Omega$$

$$\text{Z2pu} = \frac{Z_2\Omega}{Z_{2b}} = \frac{0.96}{77.44} = 0.0124\,pu$$

Note:

1. In per unit calculations involving transformer in three phase system, we follow the same principles developed for single phase system and require the base voltage on the two sides of the transformer to have the same ratio as the rated line to line voltage on the two sides of the transformer. The base kva is the same on each side.
2. To transfer the ohmic value of impedances from the voltage level on one side of 3 phase transformer to the voltage level on other, the multiplying factor is the square of the ratio of line to line voltages regardless of whether the transformer connection is

Example 1-4

The three single phase transformers each rated 25 Mva, 38.1/3.81 kv are connected as shown in figure with a balanced load of 0.6 Ω - connected resistors. Choose a base of 75 Mva, 66 kv for the high tension side of transformer and specify the base for the low tension side. Determine the per unit resistance of the load on the base for the low tension side. Then determine the load resistance referred to high tension side and the per unit and the per unit value of this resistance on the chosen base.

Solution:

(1) on the low tension side

The base for low tension side is 75 Mva, 3.81 kv

Actual value = 0.6Ω

$$\text{Base value} = \frac{(3.81)^2}{75} = 0.1935\Omega$$

Per unit value of $R_L = \frac{0.6}{0.1935} = 3.1$

(2) on the high tension side
The base for high tension side is 75 Mva, 66kv

Actual value = $0.6 \times \frac{66^2}{3.8^2 1} = 180\Omega$

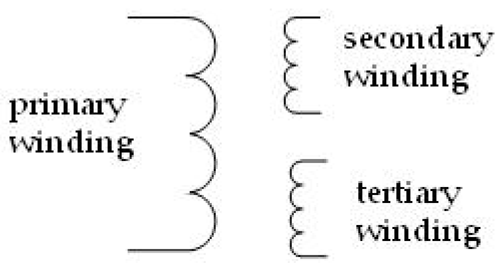
Base value = $\frac{66^2}{75} = 58.08\Omega$

Per unit value of $R_L = \frac{180}{58.08} = 3.1$

Per unit impedance of 3-winding transformers

Generally, large power transformers have three windings. The third winding is known as a tertiary winding which may be used for the following purposes

1. To supply a load at a voltage different from the secondary voltage.
2. To provide a low impedance for the flow of certain abnormal currents, such a third harmonic currents.
3. To provide for the excitation of a regulating transformer.



Note: When one winding is left open, the three winding transformer behaves as two winding transformer and standard short circuit tests can be used to evaluate per unit leakage impedances which are defined as follows

Z_{ps} = per unit leakage impedance measured from primary with secondary shorted and tertiary open.

Z_{pt} = per unit leakage impedance measured from primary with tertiary shorted and secondary open.

Z_{st} = per unit leakage impedance measured from secondary with tertiary shorted and primary pen.

$$Z_{ps} = Z_p + Z_s \qquad \text{……………(a)}$$

$$Z_{pt} = Z_p + Z_t \qquad \text{………………(b)}$$

$$Z_{st} = Z_s + Z_t \qquad \text{………………(c)}$$

Where Z_p, Z_s, and Z_t : the impedances of primary, secondary and tertiary.
Solving these equations we find

$$Z_p = ½ (Z_{ps} + Z_{pt} - Z_{st}) \qquad \text{……..(d)}$$

$$Z_s = ½ (Z_{ps} + Z_{st} - Z_{pt}) \qquad \text{……….. (e)}$$

$$Z_t = ½ (Z_{pt} + Z_{st} – Z_{ps}) \qquad \text{………..(f)}$$

These equations can be used to evaluate the per unit series impedances Z_p, Z_s, and Z_t of three winding transformer equivalent circuit from the per unit impedances Z_{ps}, Z_{pt} and Z_{st} which in turn are determined from short circuit tests.

Note.

The impedances Z_p, Z_s, and Z_t of the three windings are connected in star .

Example1-5

The 3 phase rating of 3 winding transformer are:
Primary: Star-connected, 66KV ,15MVA.
Secondary: Star-connected ,13.2KV, 10MVA
Tertiary: Delta- connected ,2.3KV,5MVA.
Neglecting resistance, the leakage impedances are

Z_{ps} = 7% on 15 Mva, 66 kv base
Z_{pt} = 9% on 15 Mva, 66 kv base
Z_{st} = 8% on 10 Mva, 13.2 kv base

Find the per unit impedances of the star connected equivalent circuit for a base of 15 Mva, 66 kv in the primary circuit

Solution:
With the base 15 Mva, 66 kv
S_{base} = 15 Mva for all three terminals is the same and
V_{b1} = 66 kv
V_{b2} = 13.2 kv
V_{b3} = 2.3 kv

Z_{ps} = 0.07 (no change)
Z_{pt} = 0.09 (no change)
Z_{st} = 0.08 ×(15/10)=0.12

$$Z_p = \frac{1}{2}(j0.07 + j0.09 - j0.12) = j0.02\,pu$$

$$Z_s = \frac{1}{2}(j0.07 + j0.12 - j0.09) = j0.05\,pu$$

$$Z_t = (j0.09 + j0.12 - j0.07) = j0.07\,pu$$

Impedance and Reactance Diagrams

Let use take a sample power system network as shown in figure below

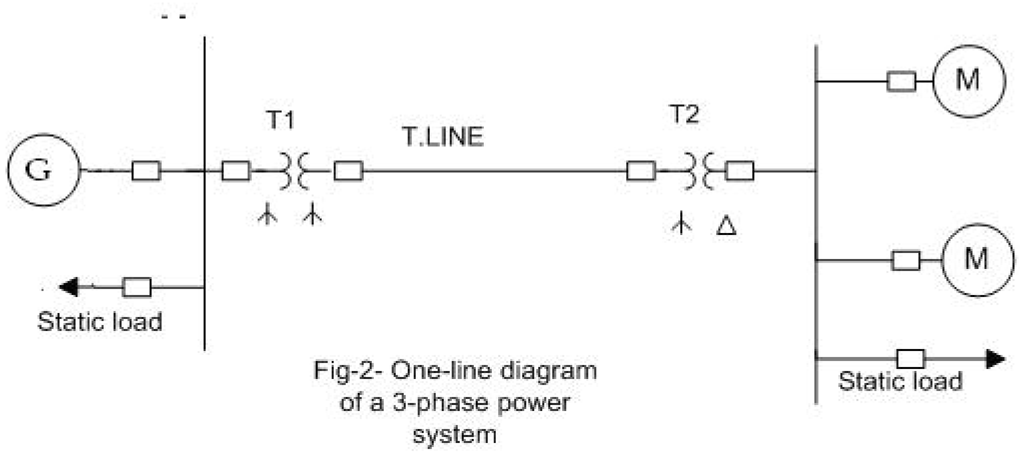
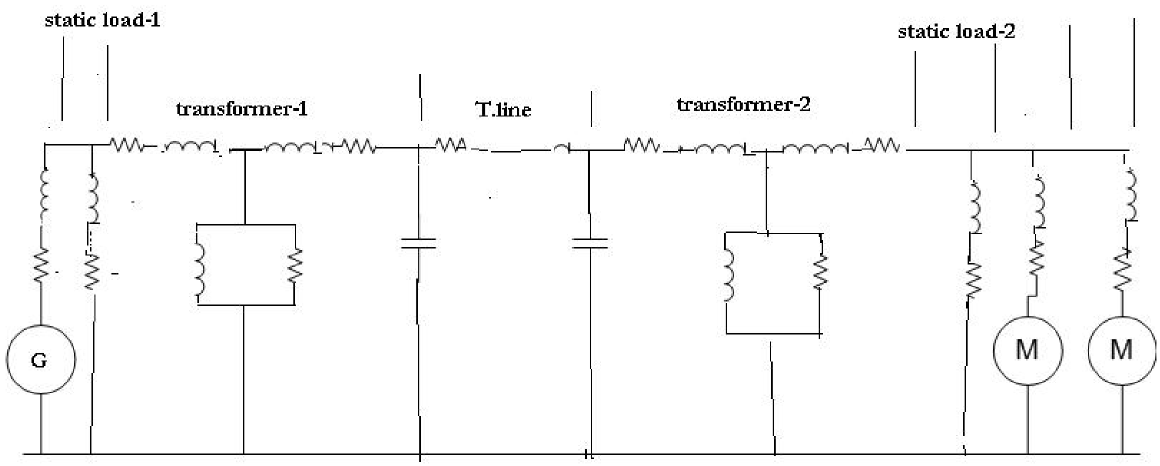


Fig-2- One-line diagram of a 3-phase power system

The impedance diagram of this sample network is shown in figure 4



In many studies, like faults calculations study, in order to simplify the calculations we can neglect all static loads, all resistances, the magnetizing current of each transformer and the capacitance of transmission line and thus we obtain the reactances diagram as shown in figure.

Notes: The impedance and reactance diagrams are sometimes called the positive sequence diagrams.